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Unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity

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A B S T R A C T

The aim of this paper is to present the unsteady boundary layer flow and heat transfer of a fluid towards a porous stretching sheet. Fluid viscosity and thermal diffusivity are assumed to vary as linear functions of temperature. Using similarity solutions partial differential equations corresponding to the momentum and energy equations are converted into highly non-linear ordinary differential equations. Numerical solutions of these equations are obtained with the help of shooting method. It is noted that due to increase in unsteadiness parameter, fluid velocity decreases up to the crossing over point and after this point opposite behaviour is noted. The temperature decreases significantly in this case. Fluid velocity decreases with increasing temperature-dependent fluid viscosity parameter (i.e. with decreasing viscosity) up to the crossing over point but increases after that point and the temperature decreases in this case. Due to increase in thermal diffusivity parameter, temperature is found to increase.

1. Introduction

The study of hydrodynamic flow and heat transfer over a stretching sheet has gained considerable attention due to its applications in industries and important bearings on several technological processes. Crane [1] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [2], Chen and Char [3], Dutta et al. [4] extended the work of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations.

All the above mentioned studies confined their discussions by assuming uniformity of fluid viscosity. However, it is known that the physical properties of fluid may change significantly with temperature. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so rate of heat transfer at the wall is also affected. Therefore, to predict the flow behaviour accurately, it is necessary to take into account the viscosity variation for incompressible fluids.

Gary et al. [5] and Mehta and Sood [6] showed that, when this effect is included the flow characteristics may change substantially compared to constant viscosity assumption. Recently Mukhopadhyay et al. [7] investigated the MHD boundary layer flow with variable fluid viscosity over a heated stretching sheet.

All of the above mentioned studies were restricted to the steady state conditions. The transient or unsteady aspects become interesting in certain practical problems where the motion of the stretched surface may start impulsively from rest. Elbashbeshy and Bazid [8] and Sharidan et al. [9] presented similarity solutions for unsteady flow and heat transfer over a stretching surface.

The present work deals with unsteady fluid flow and heat transfer over a stretching sheet in presence of wall suction. Fluid viscosity and thermal diffusivity are assumed to vary as linear functions of temperature. Similarity variable and similarity solutions are obtained and using them, a third order and a second order ordinary differential equations corresponding to momentum and energy equations are derived. These equations are solved numerically using shooting method. The effects of different parameters (viz. unsteadiness, temperature-dependent fluid viscosity, variable thermal diffusivity and suction) on velocity and temperature fields are investigated and analysed with the help of their graphical representations.

2. Equations of motion

We consider unsteady two-dimensional forced convection flow of a viscous incompressible fluid past a heated stretching sheet immersed in a porous medium in the region $y > 0$ and moving with non-uniform velocity $U(x,t) = \frac{cx}{1-\alpha t}$ where c, α are positive constants with dimensions $(\text{time})^{-1}$, c is the initial stretching rate and $\frac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time. In case of polymer extrusion, the material properties of the extruded sheet may vary with time. Here, the stretching surface is subjected to such amount of tension which does not alter the structure of the porous material.

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Nomenclature

A	fluid viscosity variation parameter
f	non-dimensional stream function
f'	first order derivative with respect to η
f''	second order derivative with respect to η
f'''	third order derivative with respect to η
M	unsteadiness parameter
Pr	Prandtl number
p,q	variables
S	suction parameter
T	temperature of the fluid
T_w	temperature of the wall of the surface
T_∞	free-stream temperature
u, v	components of velocity in x and y directions
z	variable

Greek symbols

β	thermal diffusivity parameter
η	similarity variable
k	the non-uniform value of coefficient of thermal diffusivity
μ	dynamic viscosity
μ^*	reference viscosity
v^*	reference kinematic viscosity
ψ	stream function
ρ	density of the fluid
θ	non-dimensional temperature
θ'	first order derivative with respect to η
θ''	second order derivative with respect to η

The temperature of the sheet is different from that of the ambient medium. The fluid viscosity is assumed to vary with temperature while the other fluid properties are assumed constants.

The continuity, momentum and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right), \tag{3}$$

where u and v are the components of velocity, respectively, in x and y directions, T is the temperature, κ is the coefficient of thermal diffusivity (dependent on temperature), c_p is the specific heat, ρ is the fluid density (assumed constant), μ is the coefficient of fluid viscosity (dependent on temperature), k is the permeability of the porous medium.

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u=U(x,t),\quad v=v_w(t),\quad T=T_w(x,t)\qquad \text{at } y=0, \tag{4}$$

$$u\to 0,\quad T\to T_\infty\qquad \text{as } y\to\infty, \tag{5}$$

where $v_w(t)=-v_0\sqrt{\frac{1}{1-\alpha t}}$ is the velocity of suction ($v_0>0$) of the fluid, $T_w(x,t)=T_\infty+\frac{1}{2}T_0\mathrm{Re}_x(1-\alpha t)^{\frac{-1}{2}}$ is the wall temperature [10] where $\mathrm{Re}_x=\frac{Ux}{v^*}$ is the local Reynolds number based on the stretching velocity U, T_0 is a reference temperature such that $0\leqslant T_0\leqslant T_w$ and v^* is the kinematic viscosity of the ambient fluid. The expressions for $U(x,t)$, $T_w(x,t)$, $v_w(t)$ are valid only for time $t<\alpha^{-1}$ unless $\alpha=0$.

It is to be noted that though the velocity and temperature are time dependent (initially), no initial condition is needed in the boundary as the transformed equations [see (9) and (10)] and the boundary conditions [see (11) and (12)] are independent of "t" (see Elbashbeshy and Bazid [8], Andersson et al. [10]). On the other hand if the initial and boundary conditions are taken as [instead of (4) and (5)]

$$t<0:\quad u=0,\quad T=T_\infty\qquad \text{for any } x,y, \tag{4a}$$

$$t\geqslant 0:\quad u=U(x,t),\quad v=v_w(t),\quad T=T_w(x,t)\qquad \text{at } y=0, \tag{4b}$$

$$u\to 0,\quad T\to T_\infty\qquad \text{as } y\to\infty \tag{5}$$

then also these conditions (4) and (5) reduce to Eqs. (11) and (12).

2.2. Method of solution

We now introduce the following relations for u, v and θ as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}\quad\text{and}\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

where ψ is the stream function.

The temperature-dependent fluid viscosity is given by [7]

$$\mu=\mu^*[a+b(T_w-T)], \tag{7}$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, b are constants and $b(>0)$.

We have used viscosity–temperature relation $\mu=a-bT$ ($b>0$) which is in perfect harmony with the relation $\mu=e^{-aT}$ [11] when second and higher order terms neglected in the expansions.

The variation of thermal diffusivity with the dimensionless temperature is written as

$$\kappa=\kappa_0(1+\beta\theta). \tag{8}$$

β is a parameter which depends on the nature of the fluid, κ_0 is the value of thermal diffusivity at the temperature T_w.

We introduce

$$\eta=\sqrt{\frac{c}{v^*(1-\alpha t)}}y,\quad \psi=\sqrt{\frac{v^*c}{(1-\alpha t)}}xf(\eta),$$

$$T=T_\infty+T_0\left[\frac{cx^2}{2v^*}\right](1-\alpha t)^{\frac{-3}{2}}\theta(\eta).$$

With the help of the above relations, the governing equations finally reduce to

$$M\left(\frac{\eta}{2}f''+f'\right)+f'^2-ff''=-A\theta'f''+(a+A)f'''-A\theta f''', \tag{9}$$

$$\frac{M}{2}\eta\theta'+\frac{3}{2}M\theta+2f'\theta-f\theta'=\frac{1}{\mathrm{Pr}}(\beta\theta'^2+\theta''+\beta\theta\theta''), \tag{10}$$

where $M=\frac{\alpha}{c}$ is the unsteadiness parameter, $A=b(T_w-T_\infty)$ is the temperature-dependent viscosity parameter, $v^*=\frac{\mu^*}{\rho}$.

The boundary conditions (4) and (5) then become

$$f'=1,\quad f=S,\quad \theta=1\qquad \text{at } \eta=0, \tag{11}$$

$$\text{and } f'\to 0,\quad \theta\to 0\qquad \text{as } \eta\to\infty, \tag{12}$$

where $\mathrm{Pr}=\frac{v^*\rho c_p}{\kappa}=\frac{\mu^* c_p}{\kappa}$ is the Prandtl number, $S=\frac{v_0}{\sqrt{v^*c}}$, $S>0$ corresponds to suction.

3. Numerical method for solution

The above Eqs. (9) and (10) along with the boundary conditions are solved by converting them to an initial value problem. We set

$$f' = z, \quad z' = p, \quad \theta' = q,$$

$$p' = \left(M\frac{\eta}{2}p + Mz + z^2 - fp + Apq\right)/(a + A - A\theta), \tag{13}$$

$$q' = \left[\Pr\left(\frac{M}{2}\eta q + \frac{3}{2}M\theta + 2z\theta - fq\right) - \beta q^2\right]\Big/(1+\beta\theta) \tag{14}$$

with the boundary conditions

$$f(0) = S, \quad f'(0) = 1, \quad \theta(0) = 1. \tag{15}$$

In order to integrate (13) and (14) as an initial value problem we require a value for $p(0)$, i.e. $f''(0)$ and $q(0)$, i.e. $\theta'(0)$ but no such values are given in the boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. We compare the calculated values for f' and θ at $\eta = 10$ (say) with the given boundary condition $f'(10) = 0$ and $\theta(10) = 0$ and adjust the estimated values, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution. Different values of η (such as $\eta = 6, 8, 9, 10$, etc.) are taken in our numerical computations so that numerical values obtained are independent of η chosen. We take the series of values for $f''(0)$ and $\theta'(0)$, and apply the fourth order classical Runge–Kutta method with different step sizes ($h = 0.01, 0.001$, etc.) so that the numerical results obtained are independent of h. The above procedure is repeated until we get the results up to the desired degree of accuracy, 10^{-7}.

4. Results and discussions

In order to analyse the results, numerical computations have been carried out for various values of different parameters viz. unsteadiness (M), temperature-dependent fluid viscosity (A), suction (S) and variable thermal diffusivity (β). To show the validity and the convergence of the numerical solution, numerical computations are carried out for two different step sizes (viz. $h = 0.005, 0.01$) and the errors are calculated and presented in Table 1 (taking $a = 1, B = 1, M = 0.1, S = 0.1, \beta = 1$ and $\Pr = 0.5$).

For illustrations of the results, numerical values are plotted in Figs. 1–8. In all figures, the value of 'a' is kept fixed to 1.

Figs. 1 and 2 exhibit the velocity and temperature profiles, respectively, for variable unsteadiness parameter M ($M = 0, 0.2, 0.4$). A crossing over point appears in Fig. 1. This is a special point where all the velocity curves cross each other, i.e. the velocity profiles exhibit different behaviour before and after this point. It is noticed that as the unsteadiness parameter increases, fluid velocity decreases up to the crossing over point (at $\eta = 2$) and increases after this (Fig. 1) whereas the temperature is found to decrease with the increasing unsteadiness parameter (Fig. 2). It is noteworthy that the impact of M on temperature profiles is more pronounced than on the velocity profiles in Fig. 1. Rate of heat transfer increases with increasing M.

The physical explanation behind this is as follows:

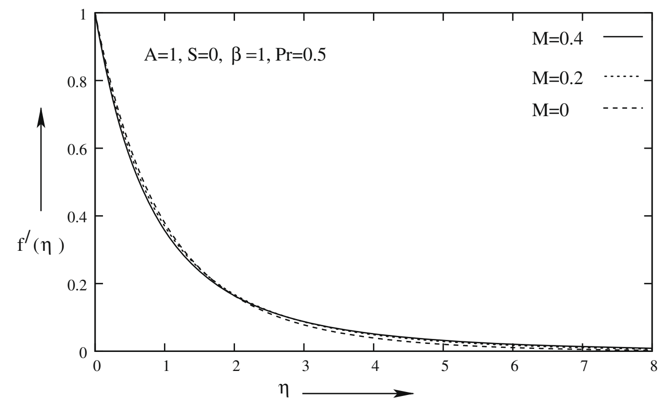

Fig. 1. Velocity profiles for variable values of unsteadiness parameter M.

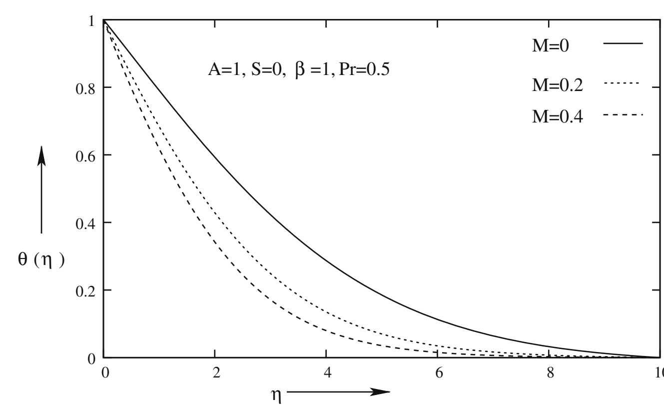

Fig. 2. Temperature profiles for variable values of unsteadiness parameter M.

Fluid velocity decreases with increasing unsteadiness parameter whereas increases with increasing temperature-dependent fluid viscosity parameter. Fig. 1 exhibits the velocity profiles for variable unsteadiness parameter (M) in presence of temperature-dependent fluid viscosity parameter ($A = 1$). At a place very near the wall, fluid velocity decreases with the increasing value of η at a slower rate in presence of non-uniform fluid viscosity (i.e. when $A \neq 0$). This is because as the parameter A increases (from $A = 0$), the fluid viscosity decreases resulting an increase of the boundary layer thickness. So near the wall, fluid velocity decreases with the increasing unsteadiness parameter (M) [the effect of unsteadiness parameter is stronger (before the crossing over point)]. But after this point velocity decreases at a slower rate and opposite behaviour is noticed as here the effect of temperature-dependent viscosity parameter (A) is stronger (after the crossing over point.

Figs. 3 and 4 display the nature of velocity and temperature profiles, respectively, for various values of temperature-dependent fluid viscosity parameter A ($A = 0, 1, 2$). Fluid velocity increases with increasing A at a particular value of η except very near the wall as well as far away from the wall (at $\eta = 10$). This means that

Table 1
Values of velocity gradient $[-f''(\eta)]$ and temperature gradient $[-\theta'(\eta)]$ for different step sizes with $a = 1, A = 1, S = 0.1, M = 0.1, \beta = 0$ and $\Pr = 0.5$.

Step size (h)	$[-f''(\eta)]$			$[-\theta'(\eta)]$		
	$\eta = 2$	$\eta = 5$	$\eta = 8$	$\eta = 2$	$\eta = 5$	$\eta = 8$
0.005	0.1195853	0.0131691	0.0026097	0.2163814	0.0557802	0.0081386
0.01	0.1195846	0.0131685	0.0026091	0.2160695	0.0556169	0.0081117
Error	0.0000007	0.0000006	0.0000006	0.0003119	0.0001633	0.0000269

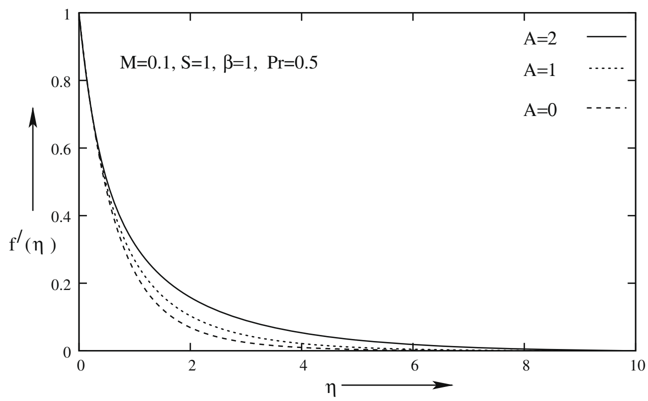

Fig. 3. Velocity profiles for variable values of viscosity variation parameter *A*.

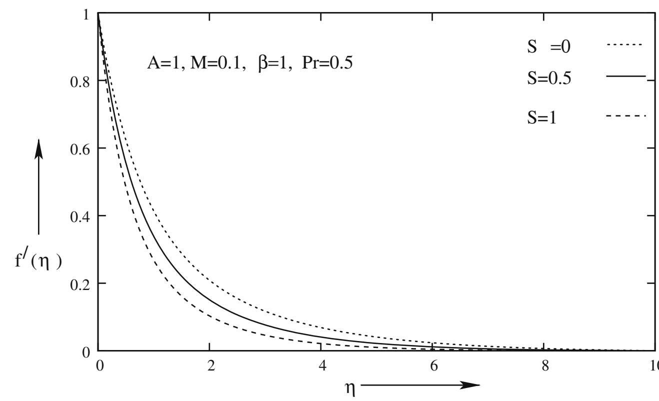

Fig. 5. Velocity profiles for variable values of suction parameter *S*.

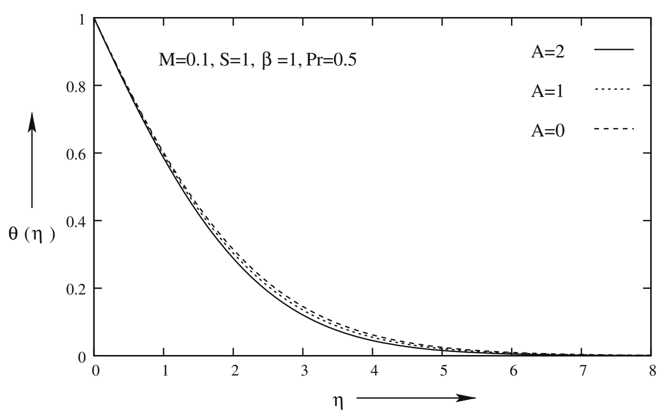

Fig. 4. Temperature profiles for variable values of viscosity variation parameter *A*.

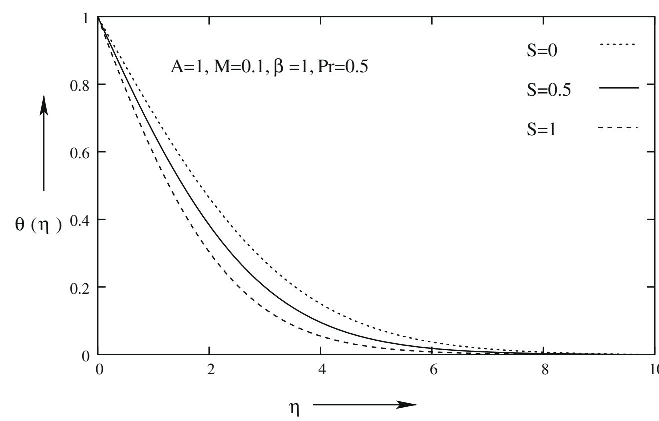

Fig. 6. Temperature profiles for variable values of suction parameter *S*.

the velocity decreases (with the increasing value of η) at a slower rate with the increase of *A* at very near the wall as well as far away from the wall. The physical explanation is as follows:

As the parameter *A* increases, the fluid viscosity decreases resulting an increment of the boundary layer thickness.

In Fig. 4, variations of temperature field $\theta(\eta)$ with η for several values of *A* are shown. This figure exhibits that the temperature decreases with the increasing value of *A*. With increasing *A*, the thermal boundary layer thickness decreases, which in turns causes to decrease the temperature profile $\theta(\eta)$.

Decrease in $\theta(\eta)$ means a decrease in the velocity of the fluid particles. So, in this case the fluid particles undergo two opposite forces: one causes to increase the fluid velocity due to decrease in the fluid viscosity (with increasing *A*) and the other causes to decrease the fluid velocity due to decrease in temperature $\theta(\eta)$ (since θ decreases with increasing *A*). Near the surface, as the temperature θ is high, the first force dominates and at far away from the surface, θ is low and so the second force dominates here.

Now we concentrate on the velocity and temperature distribution for the variation of suction parameter *S* in presence of *A*.

Fig. 5 reveals that with increasing suction ($S > 0$), fluid velocity is found to decrease, i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. This effect acts to decrease the wall shear stress. Increase in suction causes progressive thinning of the boundary layer. Fig. 6 exhibits that the temperature $\theta(\eta)$ in boundary layer also decreases with the increasing *S* ($S > 0$). The thermal boundary layer thickness decreases with *S* which causes an increase in the rate of heat transfer. Same behaviour (due to suction) is also observed in case of uniform viscosity.

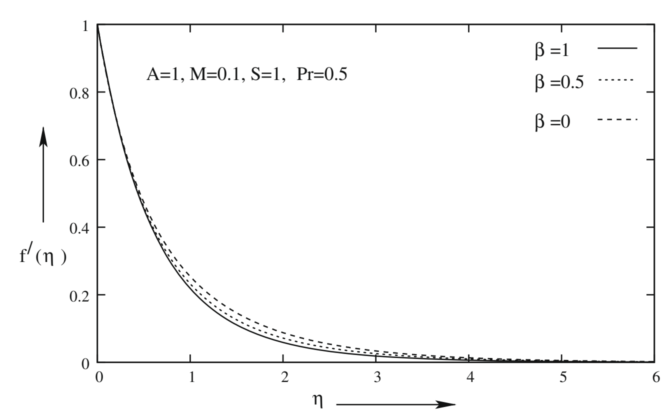

Fig. 7. Velocity profiles for variable values of thermal diffusivity parameter β.

The effects of thermal diffusivity parameter on velocity and temperature field are given in Figs. 7 and 8. Fig. 7 presents the fluid velocity profile for several values of β ($\beta = 0, 0.5, 1$). Fluid velocity decreases with the increasing values of β (Fig. 7) whereas the temperature at a particular point of the sheet increases with increasing values of β (Fig. 8). This is due to thickening of the thermal boundary layer as a result of increasing thermal diffusivity. So the fluid velocity decreases in this case. $\beta = 0$ gives the result in case of uniform thermal diffusivity.

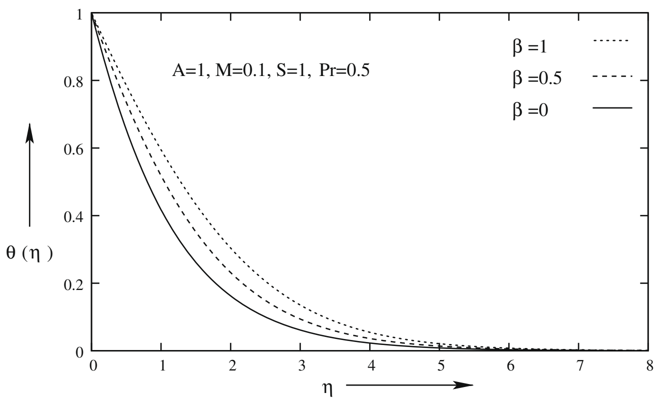

Fig. 8. Temperature profiles for variable values of thermal diffusivity parameter β.

5. Conclusion

The present study gives the solutions for unsteady boundary layer flow and heat transfer over a stretching surface with variable fluid viscosity and thermal diffusivity in presence of wall suction. Fluid velocity decreases up to the crossing over point due to increase in unsteadiness parameter. After the crossing over point, fluid velocity is found to increase with increasing unsteadiness parameter. But the temperature decreases significantly in this case. The effect of increasing temperature-dependent fluid viscosity parameter on a viscous incompressible fluid is to increase the flow velocity which in turn causes the temperature to decrease. The results pertaining to the present study indicate that due to suction the fluid velocity decreases while the rate of heat transfer increases. The temperature in the boundary layer decreases due to suction. Fluid velocity decreases but the temperature increases with increasing thermal diffusivity parameter.

It is hoped that, with the help of our present model, the physics of flow over the stretching sheet may be utilized as the basis of many engineering and scientific applications. The results derived from the present study may be useful for different model investigations.

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